(a) How many ways can 13 books be placed on 4 distinguishable shelves if the books are indistinguishable copies of the same title? (b) Suppose you have 12 unique books and 4 shelves and you want to put 3 books on each shelf. How many ways can you do that? (c) What if we had to place 12 books on 3 shelves and you could put any number of books on any shelf! And the books are all still unique. How many ways can you do that?

|  |  |  |  |
| --- | --- | --- | --- |
| subproblem | Books | Shelves |  |
| a | 13, all same book | 4 different shelves | 0-13 books on each shelf |
| b | 12 unique books | 4 different shelves | 3 books on each shelf |
| c | 12 unique books | 4 different shelves | 0-12 books on each shelf |

a) To calculate the number of ways to arrange the indistinguishable books on the shelves, you should recognize that this is how many ways there are to select 13 items from a set with 4 types of elements (the 13 items are like the "stars" and the 4 types will make up the bars) C(n+r-1,n) or Combination(13+4-1, 13)= 560.

b) To figure out how many ways we can arrange 12 unique books on 4 different shelves when you must have 3 books on each shelf, begin in this way. Step 1) what would a valid arrangement look like?

ABC\DEF\KLM\IJH

OR

CDK\ABE\LJI\FGH

Step 2) So what is a “combinatorical method” for arranging these books. How many spots are there for books? Shelf A (1,2,3), shelf B (1,2,3), shelf C(1,2,3) and shelf D (1,2,3) so that’s 12 spots for 12 books. Sort of like

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And there are 12 places to put book 1 and 11 places to put book 2 and …

Step 3) using the method I described above, could I create the same layout twice (and thus over count). No, order matters here.

So the answer is 12! (I’m not excited about the answer, this is 12 factorial)

c) To place 12 different books on 4 different shelves when you can have any number of books on a shelf, Step 1) What do valid arrangements look like? Here’s one:

No books on shelf A. Book 2, Book 7, Book 12 on Shelf B in that order. Book 1, Book 3, Book 8, Book 4 and Book 5 on Shelf C in that order. Book 6, Book 8, Book 9, Book 11, book 10 on Shelf D in that order.

Step 2) How do I build an arrangement? This is tricky.

I could figure out how many books go on each shelf first. That would be a 12-combination of 4 types of items. Why? Because we have 4 shelves (the types of items) and 12 books (imagining they are all the exact same for a minute) to put on them. One arrangement may look like this:

\*\*|\*\*\*\*||\*\*\*\*\*\* which is 2 books on shelf A, 4 books on shelf B, no books on shelf C and 6 books on shelf D. Calculating all divisions for the shelves is C(12+4-1,12). But WHICH books?! Arranging 12 books into the 12 open slots for books is 12! Don’t forget to make sure you didn't over count. Each choice of how many books to put on a shelf will be unique because C(12+4-1,12) counts each possible arrangement of books once. Therefore we would have C(12+4-1,12)\*12!

Another way would be to imagine the shelves just sitting there empty. Where can I put book one? On any of the 4 shelves. 4 choices. Now where could I put book 2? On any of the three empty shelves or to the left of Book 1 or to the right of book 1. 5 choices. Now where could I put book 3? For book 1 there were 4 choices; book 2-- 5 choices, book n has n+3 choices. Therefore we would have 4\*5\*…\*(n+3). Here n is the number of books we are placing, 12. 4\*5\*6\*…\*15 = P(15,12).